Consumption and Savings with Unemployment Risk: Implications for Optimal Employment Contracts

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Abstract

This paper derives analytically lifetime consumption and asset profiles when there are employment and unemployment risks. Without perfect insurance, consumption rises during employment and falls during unemployment, with a consequent rise in the probability of leaving unemployment. Optimal employment contracts smooth consumption during employment without causing moral hazard, by offering severance compensation. A pre-announced delay in dismissal when the job becomes unproductive provides further insurance but because of moral hazard it is not perfect. Consumption falls during delayed dismissal and there is search on the job. No delays in dismissal are offered if the level of exogenous unemployment compensation is sufficiently high.

Employment contracts often contain provisions for the payment of severance compensation to dismissed employees, or for delays in dismissals. The most common procedure that delays dismissal is the requirement to give a notice of fixed duration before dismissal. There are, however, other procedures. In many countries, minimum levels of severance compensation and dismissal delays are written in employment laws but private contracts contain similar, if not more, stringent requirements. The OECD (1999) reports that on average in its member countries employers are required to give minimum advance notice of dismissal of 1.6 months to employees of four years standing and to pay severance compensation of four weeks' wages.[1] The purpose of this paper is to in-

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[1] Provisions are more stringent in Europe than elsewhere but even in the United States, where legal provisions are virtually non-existent, similar arrangements are found in private contracts. For example,

vestigate the theoretical foundations for the existence of such provisions in employment contracts.

I study a situation in which a principal, the firm, chooses the employment contract that minimizes the cost of its job offer, which has to be acceptable to an agent, the worker. I do not attempt to justify the inclusion of severance compensation or dismissal delays in legislation but investigate whether they can be a part of an optimal employment contract. The main result of the paper is that if workers cannot insure against the risk of unemployment - the risk of both becoming unemployed and the risk associated with an uncertain duration of search - severance compensation and dismissal delays provide second-best alternatives that avoid the moral hazard of first-best insurance. The payment of severance compensation is a perfect substitute for insurance against the risk associated with an uncertain duration of employment (I refer to this as the *employment risk*). Giving advance notice before dismissal provides additional insurance against the uncertain duration of unemployment (the *unemployment risk*) by spreading income from work over a spell of uncertain duration, during which the worker searches for another job. Dismissal delays, however, are not a perfect substitute for insurance against the unemployment risk, because the failure of the firm to monitor the search strategies of its employees introduces moral hazard.

A dismissal delay is counter-intuitive in the following sense. Imagine a firm with a job that has become unproductive. It has an agreement with the worker not to fire her without giving advance notice; it is required to give notice before termination, which leads to an expected duration of unproductive employment of d periods. Keeping open an unproductive job is a nuisance for the firm and costs the worker unemployment compensation, which is subsidized by the state. The firm offers the worker instead d periods' worth of wages as severance compensation and fires her immediately. Both firm and worker are better off: the intuition is that delaying dismissal cannot be better than paying severance compensation and dismissing the worker without delay.

In this paper I show why this intuition is wrong. A worker who is given notice of dismissal begins search on the job for another job. If the expected duration of on-the-job search is d periods (bearing in mind the maximum defined by the length of notice), the

the OECD reports that in a survey conducted in 1992, it was found that between 15 and 35 percent of employees in the United States were covered by company severance plans, depending on company size. Civil rights laws and other legislation are also said to be contributing to delays in dismissals. See OECD (1999, p. 58).

expected wage cost to the firm from giving notice is dw. A risk averse worker prefers to be given advance notice and remain employed for a wage w per period, at an expected cost to the firm of dw, than be paid dw and fired immediately. Although I do not formulate the problem as the choice of optimal wage payments, it is the fact that payments to employees on notice of dismissal are contingent on the outcome of an uncertain event that gives more insurance value to dismissal delays and introduces them in optimal contracts.[2]

I show that if it is optimal to delay dismissal when a job-worker match becomes unproductive, the firm structures its compensation package in such a way as to give incentives to the worker to search on the job and quit. If the firm can monitor search effort the optimal compensation package is one that equates lifetime utility in all states of nature and periods, whilst the mismatched worker searches optimally on the job. But without monitoring of search effort the firm induces the worker to search on the job and quit, by offering a package that implies that when the job becomes unproductive the utility of remaining employed falls, and becomes progressively worse as the duration of employment becomes longer. With a perfect capital markets (a maintained assumption in this paper) there is a number of different ways that the firm can ensure that the utility of remaining employed falls with duration. One possible package holds the wage rate constant for a finite length of time and offers severance compensation to quitting workers. The worker is fired if she is still employed at the end of the notice period. This is the most common structure found in employment contracts that include dismissal delays. But other compensation structures give equivalent results. One such other package is to allow wages to fall monotonically with time employed after the job becomes unproductive. I discuss the "implementation" issues only briefly in the concluding section of the paper, my main focus being the description of the optimal contract.

My results on the optimal compensation package during a delay in dismissal mirror the results on the optimal time structure of unemployment compensation, especially those by Shavell and Weiss (1979), Sampson (1978) and Hopenhayn and Nicolini (1997), who show that optimal unemployment compensation declines with search duration. Their results, however, are derived for a more restrictive set of assumptions than

[2]Delaying dismissal is likely to be more attractive when the duration of unemployment is skewed, as it is in practice, than if it is symmetric, because there is a high probability that the worker will quit search for another job after a short time on notice. The results that I derive do not depend on skewness.

in this paper.[3] One of the contributions of this paper is to introduce a model of job loss and search that permits the derivation of analytical results with borrowing, lending and a concave utility function. Although the model in this paper is deliberately simplified, and ignores the aggregate implications of the employment contracts, it can easily be extended to a model of labor market equilibrium with unemployment.[4]

The features of the optimal employment contract that I derive are often collectively described in the empirical literature as "employment protection." There is a large literature on employment protection legislation, which studies partial or equilibrium models with risk neutrality in order to quantify the effect of various policy measures on employment and wages. Special emphasis is given to compulsory severance compensation and dismissal frictions.[5] The general conclusion reached in this literature is that employment protection measures do not have a significant impact on steady-state employment, but are likely to influence the dynamics of employment and wages. The main implication of the analysis of this paper for most of this literature is that a proper evaluation of employment protection measures should take into account the fact that they may be optimal responses to missing markets and this should influence the impact that they have on equilibrium.

Also related to the model of this paper is another strand of the literature, which studies the behavior of wealth and the unemployment hazard during search when there is risk aversion. Danforth (1979) shows that with decreasing risk aversion reservation wages fall and so the probability of leaving unemployment rises. A similar result is

[3] Although Shavell and Weiss (1979) allow the possibility of borrowing and lending in an extension of their model, they are unable to derive any results in this case when there is moral hazard, and their famous result holds only in the case where consumption is identically equal to income. Sampson (1978) who, like Shavell and Weiss, studies the optimal structure of unemployment compensation and reaches similar conclusions, also assumes away both borrowing and lending. Hopenhayn and Nicolini (1997) make a similar assumption and the absence of borrowing and lending also appears to be critical for some of their results (e.g. the result that the optimal insurance package should tax workers according to their unemployment history. It is difficult to see how the government could implement the contract with borrowing and lending and long horizons.

[4] Other papers on optimal unemployment insurance address different sets of issues. For example, Acemoglu and Shimer (1999) study a model with constant absolute risk aversion to derive results on the efficiency of unemployment insurance, given that risk-averse workers accept offers too quickly. Andolfatto and Gomme (1996), Costain (1995), Valdivia (1995) and Wang and Williamson (1996) study calibrated models to derive the implications of unemployment insurance for welfare and aggregate economic activity and the optimal level of UI benefits in calibrated economies..

[5] See in particular Lazear (1990). For recent summaries see Nickell and Layard (1999) and Bertola (1999) and for more recent contributions see Ljungqvist and Sargent (1998), Mortensen and Pissarides (1999), Blanchard and Wolfers (2000), and Pissarides (2001).

derived by Lentz and Tranaes (2001) for a more general model of job search, with both an employment and an unemployment risk and borrowing and lending.

It is important for the results of this paper that the firm should be better able to insure against fluctuations in income than workers are. This property, the asymmetric access to insurance markets by firms and workers, is the key assumption behind the static implicit contract theory, and this paper can be viewed as an application of the ideas first developed in that theory to dynamic search equilibrium (see Baily, 1994, Azariadis, 1975 and Gordon, 1994).

Section 1 outlines the framework used to study the implications of non-linear utility for consumption and job search. Section 2 studies the optimal consumption and search strategies when workers are paid their marginal product, and section 3 studies the other extreme, choices made under a full set of insurance contracts. Section 4 forms the core of the paper and studies first, the insurance implications of severance compensation and second the insurance implications of delayed dismissal. Section 4.3 shows that whereas it is always optimal to include severance compensation in employment contracts, whether dismissal delays are part of a contract or not depends on the subsidy received by unemployed workers and on their risk aversion. The concluding section 5 briefly summarizes the main findings and discusses issues in the implementation of the optimal contract. All proofs are collected in the Appendix.

1 Preliminaries

The model is a partial one and focuses on the relation between a risk-neutral firm that owns a job and a risk-averse worker who owns a time endowment. Time is discrete and the horizon infinite. The time endowment yields no utility but it enables the individual to hold a job. Utility is derived only from consumption, at the rate $u(c)$ per period, with $u'(c) > 0, u''(c) \leq 0$ and $u'(0) = \infty$, although there are also some lump-sum disutilities associated with holding some jobs and with changing jobs, which are specified later. There is unlimited borrowing and lending at a safe rate of interest r, which accrues during the period, and which is also used to discount future utility. I define the discount factor $\beta = 1/(1 + r)^{-1}$. The utility function, discount rates and capital structure are chosen such that under a full set of insurance contracts the consumption profile is flat in all states of nature, irrespective of the income profile. It is assumed that there are no exits from the labor force, although the introduction of insurable death a la Blanchard (1985) is a straightforward matter and would not alter the results.

The objective is to describe the features of a contract when there is a positive probability that the job will end and when the date of arrival of a new job is uncertain. I set up the model in such a way that most of the interesting questions about the design of the contract can be analyzed in the first period of the worker's life. I use throughout the following simplified framework.

There are two or more differentiated types of agents and jobs. The match between a worker and a job is good if they are of the same type and bad if they are of different types. Net output is p per period in all matches, irrespective of type, but mismatched workers forego a lump-sum utility cost in order to produce this output. Workers who are matched to a job of their type do not forego any utility to produce; they stay in it for ever and earn their marginal product p per period.

Workers do not initially know how to recognize their job type. They are born into a randomly-selected job and spend the first period of their life in productive employment. During this period they learn about their job type, and how to inspect and recognize future job types. The utility cost of a mismatch is sufficiently high that unemployment dominates production in a job of the wrong type. But the disutility is sufficiently low that all agents prefer to produce in period 1, and run the risk of mismatch, from taking leisure for ever. The probability that a worker is born into a job of her type in period 1 is a fixed $m \in (0, 1)$.

These assumptions capture the idea that there is initially learning about the quality of matches, and so turnover and employment risk are higher at short tenures (Jovanovic, 1979, Wilde, 1979). In period 1 all workers produce output p in their allocated job, learn about their job type and also learn how to recognize other job types without the need to experience them. Job types are "experience" goods in the first period but "inspection" goods in all subsequent periods. The latter assumption makes employment from period 2 onward an absorbing state and simplifies the derivations, without loss of essential generality.[6]

Workers who are in their job type in period 1 stay in it for ever, producing p per

[6]Employment is an absorbing state in all periods in Danforth (1979), Hopenhayn and Nicolini (1997) and Acemoglu and Shimer (1999) but not in Lentz and Tranaes (2001), who derive the effects of unemployment risks on savings in a more general environment.

period, but those who are not in their job type do not produce again in that job. If they find an acceptable job of their type at the end of period 1 they move to it at the beginning of period 2 and stay in it for ever, again producing p per period. If they do not find an acceptable job and their employment contract specifies dismissal (with or without severance payment) they become unemployed and search for a job of their type. If their employment contract specifies a delay before dismissal they remain employed but do not produce, and can again search for another job of their type (I refer to this state as being *on delayed dismissal* or *on notice of dismissal*). Unemployed workers receive subsidy $b < p$ and workers on notice of dismissal receive a transfer from the firm. Of course, a worker can quit at any time into unemployment and receive the subsidy b, but once she quits she cannot be rehired by the same firm, which is not of her type. The circumstances that lead an agent to make decisions among the alternative states, the factors that influence their decisions, and whether contracts specify severance compensation or delayed dismissal, are the subject of analysis in this paper.

The worker searches on the job in period 1 because of the risk of a mismatch and also searches in subsequent periods if the job in period 1 is revealed to be not of her type. There are no search costs but before accepting an offer the worker has to pay a moving cost $x \geq 0$, which differs across jobs. The cumulative distribution of x for the best job offer available to the worker each period is denoted by $G(x)$ and has support in the positive quadrant. The mobility cost is measured in utility units and it is strongly separable from the utility of consumption.

There are two income risks in this model which are insurable with a full set of insurance markets. First, the risk that productive employment in the first job lasts either one period, because of mismatch, or until the end of life. Second, conditional on mismatch in period 1, the risk that non-production (i.e., either unemployment or unproductive employment) lasts for one or more periods. The first risk is the *employment risk* and the second the *unemployment risk,* as each is associated with an uncertain duration of employment or unemployment.

The firm offers the worker an employment contract in period 1 which minimizes the cost of providing a pre-specified level of lifetime utility to the worker. I assume that workers are born with zero assets and that the firm can monitor the worker's assets for the duration of the contract. It cannot, however, monitor the worker's search effort. Therefore, the firm can act as if it can choose a consumption sequence for the worker, for the duration of the contract, subject to incentive-compatibility constraints on search effort. This is a typical principal-agent problem with moral hazard, with the firm acting as the principal who minimizes the cost of providing a consumption level to the worker. It is convenient to set up the problem as if the contract ends either at the beginning of period 2 if the worker discovers that she is of the firm's type or in period $t \geq 2$, if the worker is not of the firm's type. In the latter case, the contract ends either when the worker quits or when she is dismissed. For the duration of the contract the firm provides consumption to the worker. When the contract ends the firm makes a transfer of assets to the worker who then chooses her own consumption levels. Clearly, there is no loss of generality if I assume that the contract of well-matched workers ends at the beginning of period 2 because there is no more uncertainty attached to lifetime earnings for these workers.

2 Spot wage contracts

I derive first the lifetime consumption profile in the absence of insurance and contingent transfers from the firm to the worker. Workers receive their marginal product p when employed, and subsidy $b < p$ when unemployed. They do not receive any income if they are on notice of dismissal, making this option sub-optimal.

An equilibrium is a consumption sequence $\{c_t^s\}$ for each state and period and an acceptance rule for each period of search. The states of nature are employment $(s = j)$ or unemployment $(s = u)$, with $t = 1, 2, .., \infty$ and agents always employed in $t = 1$ but employed or unemployed in subsequent periods. The agent maximizes expected utility subject to a sequence of budget constraints and a value for initial assets, which is assumed to be zero, and subject to rational expectations about the sequences $\{p\}, \{b\}$, the distribution of costs $G(x)$ and the probability of a good period 1 match, m.[7]

Consider first an agent's maximization problem in a job of her type. A job of the worker's type is an absorbing state: income is equal to p per period until death and because productivity in all other jobs is also p, the worker has no incentive to search for another job. By assumption, the job that starts in period 1 becomes an absorbing state in period 2 with probability m and all new jobs that start from period 2 onward

[7] The utility cost of effort for the mismatched workers in period 1 is sunk and plays no role in the subsequent analysis, beyond the fact that it makes production in poor matches in the second and subsequent periods sub-optimal. I will ignore it in the modeling.

are absorbing states with probability 1.

For initial assets A_{t-1} the end-of-period budget constraint in period $t \geq 2$ for an agent in a job of her type is

$$A_{t-1} + \beta \left(p - c^j_t - A_t \right) \geq 0. \tag{1}$$

Lifetime utility in period t satisfies the Bellman equation

$$U^j(A_{t-1}) = \max_{c^j_t, A_t} \left\{ \beta \left(u(c^j_t) + U^j(A_t) \right) \right\} \tag{2}$$

and maximization gives

$$c^j_t = c^j_{t+i} = p + rA_{t-1} \quad \forall i \geq 1. \tag{3}$$

Consumption in jobs of the agent's type is constant because there is no income risk. I denote by c^j_t the flat profile in a job that starts in period $t \geq 2$, and by c^j_1 the flat consumption profile in the first job from period 2 onward, if the job is of the agent's type. Consumption in period 1 is denoted by c_1 and consumption in each period t that the agent is unemployed is denoted by c^u_t, for $t \geq 2$. The agent's value function in a job of her type, the solution to (2), is

$$U^j(A_{t-1}) = \frac{u(c^j_t)}{r} = \frac{u(p + rA_{t-1})}{r}, \qquad t \geq 2. \tag{4}$$

I consider next the maximization problem in $t \geq 2$ when the individual is unemployed with initial assets A_{t-1}. During unemployment job offers arrive in each period and the worker can move to one by bearing a one-off utility cost x, which has distribution $G(x)$. By the separability of the mobility cost and the full information on $G(x)$, job acceptance satisfies the reservation property: the individual accepts a job at the beginning of period t if the realized mobility cost $x \in [0, R_t]$, where R_t is a reservation value. I denote by $\bar{x}_t$ the expected acceptance cost conditional on the reservation R_t, i.e. $\bar{x}_t = E(x|x \leq R_t)$. It follows that the utility function in the event of unemployment in t satisfies the Bellman equation

$$U^u(A_{t-1}) = \max_{c^u_t, R_{t+1}, A_t} \left\{ \beta \left(u(c^u_t) + G(R_{t+1}) \left(U^j(A_t) - \bar{x}_{t+1} \right) + (1 - G(R_{t+1}))U^u(A_t) \right) \right\} \tag{5}$$

The end-of-period budget constraint in period t is

$$A_{t-1} + \beta \left(b - c^u_t - A_t \right) \geq 0. \tag{6}$$

The first order maximization conditions yield, after application of the envelope theorem,

$$u'(c_t^u) = G(R_{t+1})u'(c_{t+1}^j) + (1 - G(R_{t+1}))u'(c_{t+1}^u), \tag{7}$$

and

$$R_{t+1} = U^j(A_t) - U^u(A_t). \tag{8}$$

Consider finally the maximization program in period 1. Income is equal to p but the agent chooses consumption with uncertainty about the lifetime income path. The Bellman equation satisfied by lifetime utility at birth is

$$U = \max_{c_1, R_2, A_1} \left\{ \beta \left[u(c_1) + mU^j(A_1) + (1 - m)\bar{U}(A_1) \right] \right\}, \tag{9}$$

where $\bar{U}(A_1)$ is the expected lifetime utility when the job in period 1 is not of the worker's type. In this event the agent either moves to another job of her type, with (conditional) probability $G(x)$, or becomes unemployed. Therefore

$$\bar{U}(A_1) = G(R_2)(U^j(A_1) - \bar{x}_2) + (1 - G(R_2))U^u(A_1). \tag{10}$$

If the initial job is of the worker's type consumption from period 2 onward is the same as in a new job of her type, because the value of initial assets is A_1 and income is p per period in both jobs. Therefore the $U^j(A_1)$ in (9) and (10) are the same.

The budget constraint in period 1, given zero initial assets, is

$$p - c_1 - A_1 \geq 0. \tag{11}$$

The necessary and sufficient maximization conditions satisfy

$$u'(c_1) = (m + (1 - m)G(R_2))\, u'(c_2^j) + (1 - m)(1 - G(R_2))u'(c_2^u). \tag{12}$$

$$R_2 = U^j(A_1) - U^u(A_1). \tag{13}$$

It follows from (12) and (7) that both the employment and the unemployment risk give rise to a lifetime consumption profile that is not flat. I now show (see the Appendix for proof)

Proposition 1 *The employment risk causes a rising consumption profile and the unemployment risk a falling consumption profile.*

The optimal policy is one where the agent consumes c_1 in the first period and increases her consumption permanently to a higher level if the job turns out to be of her type, or if she moves to another job of her type. If the job is not of her type and becomes unemployed in period 2 she reduces her consumption. During search consumption falls and when a job is found it rises to a permanently higher level. This policy also implies

Proposition 2 *If the value function is a concave function of beginning-of-period assets, asset holdings fall during unemployment and the probability of leaving unemployment rises.*

Concavity of the value function is not, however, guaranteed. Differentiating twice the Bellman equation (5) for any $t \geq 2$, we obtain

$$\begin{aligned} U^{u\prime\prime}(A_{t-1}) &= g(R_{t+1}) \left(U^{j\prime}(A_t) - U^{u\prime}(A_t) \right)^2 \\ &+ \left[G(R_{t+1}) U^{j\prime\prime}(A_t) + (1 - G(R_{t+1})) U^{u\prime\prime}(A_t) \right] \frac{\partial A_t}{\partial A_{t-1}}. \end{aligned} \tag{14}$$

The first term on the right-hand side is positive, so the concavity of the utility function does not guarantee a concave value function. Local non-concavities, if they exist, imply that the introduction of lotteries increases welfare. As Lenz and Tranaes (2001) show, lotteries effectively make the value function concave and guarantee the declining wealth during search. But as both Lenz and Tranaes and Hopenhayn and Nicolini (1997) also demonstrate, in calibrations with reasonable parameter values and utility functions the value functions are always concave.[8] The explicit introduction of lotteries complicates the analysis - through the introduction of another choice margin - and essentially makes the value function linear over its non-concave range. The results obtained are qualitatively the same as the results obtained when the value function is concave, when the lotteries become redundant. For these reasons, and in light of the results of Lenz and Tranaes and Hopenhayn and Nicolini, I will not introduce explicit lotteries but derive results only for the parameter ranges that imply a concave value function.[9]

[8] Intuitively, non-concavities require a large $u'(c_t^j) - u'(c_t^u)$ but a small $u''(c_t^j) - u''(c_t^u)$ and a large frequency $g(R_{t+1})$, which are implausible.

[9] Lenz and Tranaes (2001) introduce explicitly the lottery option and derive the declining wealth profile under general conditions. Hopenhayn and Nicolini (1997) follow the approach that I follow here and derive results for the range of parameters that are consistent with concavity. Danforth (1979) faced the same problem and derived results for the case of decreasing absolute risk aversion. A convex value function with no lottery options has implausible implications. For example, it implies that consumption declines with wealth.

The result that the probability of leaving unemployment rises during search when the value function is concave depends only on the fact that income and consumption in a job are both higher than in unemployment, and during unemployment wealth is falling. With decreasing marginal utility of consumption, the agent is more anxious to move into a job the longer she has been unemployed. The fact that assets are falling during unemployment also implies that the agent is consuming more than her "permanent income" during unemployment

$$c_t^u \geq rA_{t-1} + b, \tag{15}$$

so she dissaves on the expectation that when she finds a job income will rise and she will repay her accumulated debts. The Inada restrictions on the utility function and the argument used in the proof of proposition 1 require

$$\lim_{t\to\infty} (A_{t-1} - A_t) = 0,$$

$$\lim_{t\to\infty} c_t^u = 0.$$

(6) and (3) can then be used to derive a lower bound on asset holdings and consumption in a good job

$$\begin{aligned} \lim_{t\to\infty} A_t &= -b/r, \\ \lim_{t\to\infty} c_t^j &= p - b \end{aligned}$$

The gap between consumption in a job and consumption in unemployment necessarily increases with the duration of unemployment. The maximum gap is equal to the gap between income in a job and unemployment income.

3 Full insurance

When workers have access to actuarially fair insurance against all income risks their consumption profile becomes flat and independent of state of nature. This result emerges readily from the assumptions of constant and equal rate of interest and rate of time preference and the existence of a perfect capital market, and will not be demonstrated in full. As an illustration, consider a one-period insurance contract for workers in period 1. With a full set of insurance contracts the worker can insure against the employment risk by buying insurance that will pay her I_1 at the beginning of period 2 if she becomes unemployed. The risk of this is $(1-m)(1-G(R_2))$, and so actuarial fairness implies that the budget constraint for period 1 changes from (11) to

$$p - c_1 - A_1 - (1-m)(1-G(R_2))I_1 = 0. \tag{16}$$

At the end of period 2 initial assets if the agent is in a job are worth $(1+r)A_1$, as before, and in the event of unemployment they are worth $(1+r)(A_1+I_1)$. Because I_1 is a choice variable, the agent can use it to transfer wealth between the states of employment and unemployment so as to maintain the same consumption level in each state. With a full set of insurance contracts the state does not influence the consumption level.

This result, however, is achieved for given transition probabilities. If the insurance company cannot monitor the search or quitting behavior of the worker, the flat consumption profile will give rise to moral hazard that will lead to the breakdown of insurance against both the unemployment and employment risks. Insurance against the unemployment risk gives rise to conventional moral hazard that prevents workers from accepting job offers, of the type commonly analyzed in the unemployment insurance literature. When there is insurance condition (13) changes to

$$R_2 = U^{\mathsf{j}}(A_1) - U^{\mathsf{u}}(A_1 + I_1). \tag{17}$$

With consumption equal in all states of nature both lifetime utilities are equal to $u(\bar{c})/r$, where $\bar{c}$ is the common consumption level, giving the solution $R_2 = 0$, and the same holds in all periods t during which the agent searches for another job.

Insurance against the employment risk gives rise to a different type of moral hazard, temporary layoffs. Well-matched workers and firms can gain by colluding to separate temporarily, to enable the worker to collect the contingent claim from the insurance company. The loss to the pair from separating for one period is the marginal product p and the gain is the unemployment subsidy b and the insurance payment I_1. If $b + (1+r)I_1 > p$ this would be an optimal response to the contract, and if this is anticipated by the worker she might choose I_1 such that this condition is satisfied.[10]

[10] The moral hazard in this connection is closely related to the one discussed in the literature on temporary layoffs in the absence of perfect experience rating. Feldstein (1977) first showed how partial experience rating leads to excessive temporary layoffs, as firms and workers collude to maximize their revenue from the government subsidy to workers on layoff.

4 Optimal employment contracts

When workers have no access to insurance markets for income risk, employment contracts can make Pareto improvements by incorporating contingent transfers between risk-neutral firms and risk-averse workers. I derive the optimal contract under the following assumptions:

1. The firm can monitor the asset position of the worker, who has access to a perfect capital market (but not to insurance markets).
2. The firm cannot monitor the worker's search strategy.
3. The firm cannot make payments to unemployed workers but can make either positive or negative payments to employed workers.
4. The firm can monitor the quality of the job match.
5. The firm cannot monitor the destination of the worker after separation.

Assumptions 1 and 2 define the typical environment analyzed in the unemployment insurance literature, with the added generalization that the worker has access to a perfect capital market. Assumption 3 implies that there is no private unemployment insurance. The implications of the other two assumptions will be stated shortly.

An employment contract is *optimal* if it yields at least some exogenous utility level $\bar{U}$ to the worker at minimum cost to the firm. Employment contracts are written at the beginning of period 1. A worker who joins a firm after period 1 with initial assets A_{t-1} is necessarily in a good match and enjoys the utility level defined in (4). Assumptions 1-5 imply that an employment contract can in general be defined as: a consumption c_1 for period 1, an asset transfer A_1 from the firm to the worker in period 2 in the event of a good match,[11] a period $T \geq 1$, at the end of which the worker is dismissed in the event of a poor match, an asset transfer A_t^{j} for $t = 1, ..., T-1$ if the worker quits before dismissal, an asset transfer at dismissal A_T^{u}, and a consumption sequence during the delay in dismissal $\{c_t^{\mathsf{n}}\}$ for $t = 2, ..., T$.

Assumption 4 ensures that the worker cannot falsely declare a bad match, quit and collect A_1^{j} (or A_T^{u} if $T = 1$). This removes a potential moral hazard problem in the

[11] The worker then chooses her optimal consumption sequence under complete certainty, and enjoys the utility level defined in (4).

payment of severance compensation to dismissed employees.[12] Assumption 5 implies that the asset transfer at separation, A_T^u, is not contingent on the destination of the worker.[13]

Let now $V(U)$ be the minimum cost of the contract to the firm at the beginning of period 1. U is the lifetime utility that the contract yields to the worker. If the match is revealed to be good the firm makes a transfer A_1 to the worker at the beginning of period 2 and the contract ends. The transfer yields the worker lifetime utility $U^j(A_1)$. If it is bad the firm may offer to keep the worker but give notice of dismissal at some future date T. Let U_t^n $(t = 2, ..., T)$, be the lifetime utility of the worker at the beginning of period t when the match is bad but the worker is employed on notice of dismissal. The minimum cost of the contract to the firm at the beginning of period t is denoted $V(U_t^n)$. For $T \geq 3$, $V(U)$ satisfies the Bellman equations

$$V(U) = \beta \min \left\{ c_1 + mA_1 + (1-m) \left(G(R_2)A_1^j + (1-G(R_2))V(U_2^n) \right) \right\} \tag{18}$$

$$V(U_t^n) = \beta \min \left\{ c_t^n + G(R_{t+1})A_t^j + (1-G(R_{t+1}))V(U_{t+1}^n) \right\}, \quad t = 2, ..., T-1 \tag{19}$$

$$V(U_T^n) = \beta \min \left\{ c_T^n + A_T^u \right\}. \tag{20}$$

For $T = 2$, when there is a dismissal delay of only one period, (19) is redundant, and for $T = 1$, when there is no delay, $V(U)$ satisfies the simple equation

$$V(U) = \beta \min \left\{ c_1 + mA_1 + (1-m)A_1^u \right\}. \tag{21}$$

The contract is acceptable to the worker if it is worth at least $\bar{U}$, constraining the firm to offer

$$U \geq \bar{U}. \tag{22}$$

As with the firm's value equation, when solving for U it is convenient to distinguish contracts of length 1, 2 and at least 3 periods. For $T \geq 3$ lifetime utility satisfies:

$$U = \beta \left(u(c_1) + mU^j(A_1) + (1-m) \left(G(R_2) \left(U^j(A_1^j) - \bar{x}_2 \right) + (1-G(R_2))U_2^n \right) \right), \tag{23}$$

[12] Such conditions on the payment of severance compensation are sometimes found in practice, when the worker is paid compensation when she is fired but not when she quits against the firm's wishes.

[13] Even if the firm can monitor the worker's destination and makes transfers contingent on destination, if the transfer to workers entering unemployment is higher than the one to workers who have accepted another job, workers who find a new job can collude with the new employer to delay hiring. The worker enters in the meantime unemployment, in order to collect the severance payment. This moral hazard problem is similar to the one that gives rise to temporary layoffs and does not allow third-party insurance contracts against the employment risk.

$$U_t^n = \beta \left[u(c_t^n) + G(R_{t+1}) \left(U^j(A_t^j) - \bar{x}_{t+1} \right) + (1 - G(R_{t+1}))U_{t+1}^n \right], \quad t = 2, ..., T-1 \tag{24}$$

$$U_T^n = \beta \left[u(c_T^n) + G(R_{T+1}) \left(U^j(A_T^u) - \bar{x}_{T+1} \right) + (1 - G(R_{T+1}))U^u(A_T^u) \right]. \tag{25}$$

For $T = 2$ equation (24) is redundant, and for $T = 1$ lifetime utility becomes

$$U = \beta \left[u(c_1) + mU^j(A_1) + (1-m) \left[G(R_2) \left(U^j(A_1^u) - \bar{x}_2 \right) + (1 - G(R_2))U^u(A_1^u \right] \right]. \tag{26}$$

The fact that the firm cannot monitor the search strategy of the worker introduces one incentive-compatibility constraint for each period of on-the-job search (period 1 and each subsequent period of notice in the event of a bad match). The incentive-compatibility constraints are derived from the unconditional maximization of lifetime utility with respect to the reservation R_t. In period T, the search strategy chosen by the worker satisfies

$$R_{T+1} = U^j(A_T^u) - U^u(A_T^u). \tag{27}$$

In period $T-1$ and earlier periods the incentive-constraints are

$$R_{t+1} = U^j(A_t^j) - U_{t+1}^n, \quad t = 1, ..., T-1. \tag{28}$$

Implicit in the specification of the maximization program is the assumption that the worker will never want to quit into unemployment during the delay in dismissal, i.e. that $U_{t+1}^n \geq U^u(A_t^j)$ is not binding for the duration of the contract. This follows trivially from the fact that if it were, the firm would dismiss the worker into unemployment than keep her employed, because this would reduce the costs of the firm in future periods (the financing of consumption if the worker were to remain employed).

I first characterize the optimal contracts for any arbitrary duration $T \geq 1$, starting with $T = 1$, before fully characterizing the optimal duration. Contracts of duration 1 do not give notice before dismissal but may pay severance compensation. Contracts of longer duration delay the dismissal of unproductive employees.

4.1 Severance compensation

When $T = 1$, I say that the firm pays severance compensation when $A_1^u > A_1$. A_1^u is the transfer made to workers who are not good matches and who, by assumption, separate at the end of period 1. A_1 is the transfer to a good match who remains with the firm.

If $A_1^{\mathsf{u}} = A_1$ the employment contract contains no insurance provision. The worker could be paid a wage rate in period 1 and replicate this situation through the perfect capital market, by borrowing or lending so as to start period 2 with non-contingent assets A_1. If $A_1^{\mathsf{u}} > A_1$ the worker who separates starts period 2 with more assets than the worker who stays with the firm. The difference $A_1^{\mathsf{u}} - A_1$ is a severance compensation. The assumption that the firm can verify whether the match is good or not implies that the severance compensation acts as insurance against the risk that the match is revealed to be unproductive, but not as insurance against the risk attached to the destination of the worker.

The optimal contract satisfies minimization conditions for (21) subject to (26) and (27). Let in general λ_{t} denote the shadow price of the incentive-compatibility constraint for period t. If the firm is constrained by the incentive-compatibility constraints because it cannot monitor the worker's search strategy, $\lambda_{\mathsf{t}} \neq 0$. If it can dictate the worker's search strategy, $\lambda_{\mathsf{t}} = 0$. When $T = 1$, however, the firm dismisses the worker at the end of period 1 and in the absence of further transfers, it is not constrained by the worker's search strategy: the worker's choice of search strategy does not enter the firm's cost function, and the shadow price of (27) at the optimum is $\lambda_1 = 0$ (see the Appendix for proofs). The optimal contract then satisfies

$$u'(c_1) = U^{\mathsf{j}\prime}(A_1) = G(R_2)U^{\mathsf{j}\prime}(A_1^{\mathsf{u}}) + (1 - G(R_2))U^{\mathsf{u}\prime}(A_1^{\mathsf{u}}). \tag{29}$$

Application of the envelope theorem to lifetime utilities and substitution into (29) yields

$$u'(c_1) = u'(c_1^{\mathsf{j}}) = G(R_2)u'(c_2^{\mathsf{j}}) + (1 - G(R_2))u'(c_2^{\mathsf{u}}). \tag{30}$$

The fact that the incentive compatibility constraints are not binding in this case make severance payments a first-best insurance against the risk of a bad match (the employment risk). In the absence of contingent transfers, proposition 1 has demonstrated that the employment risk increases consumption in a good match. (30) shows that consumption in a good match is now flat: the consumption chosen in period 1, c_1, is at the level of consumption in all future periods in a good match, c_1^{j}.

But the failure of the firm to monitor the destination of the worker, or make payments after entry into unemployment, implies that severance compensation does not insure against the unemployment risk. In the event of separation because of a poor match, the environment from period 2 onward is identical to the one studies in proposition 1, when initial assets are A_1^{u}. Consumption increases permanently when the worker goes to another job and falls when she joins unemployment. Equation (30) and proposition 1 yield:

$$c_2^{\mathsf{j}} > c_1^{\mathsf{j}} = c_1 > c_2^{\mathsf{u}}. \tag{31}$$

Similarly, the optimal consumption profile in the event of unemployment of longer durations also satisfies proposition 1. When the agent finds a job consumption increases permanently to a higher level but during unsuccessful search it decreases.

It is straightforward to show that severance compensation is positive in this environment. Consumption from period 2 onward in the event of a good match and consumption in another job accepted in period 2 both satisfy (3), for initial assets A_1^{j} and A_1^{u} respectively. Hence $A_1^{\mathsf{u}} > A_1^{\mathsf{j}}$, by virtue of the first inequality in (31).

Proposition 3 *If workers separate without delay in the event of a bad match, the firm pays positive severance compensation. The consumption profile is flat in all periods in the event of a good first-period match but consumption falls if the match is bad and the worker joins unemployment, or rises if the worker moves immediately to another job.*

4.2 Delayed dismissal

Partial insurance against the unemployment risk can be offered by delaying dismissal in the event of a bad match. Delaying dismissal has insurance value because the employment period is extended and the firm can make payments contingent on the worker's state. The worker searches on the job during the delay period and so there is a positive probability that she will move to another job without entering unemployment. During this period the firm can effectively monitor the worker's destination, because if the worker quits, it will be to take another job. It can therefore make payments conditional on destination and so increase the insurance value of its contract.

The disadvantage of delaying dismissal is that the worker cannot claim the unemployment subsidy during delay. One other potential cost and one other benefit of delayed dismissal are ignored in the analysis that follows, without loss of essential generality. If the job is costly to maintain the firm suffers losses by delaying dismissal, which can be avoided if the worker is fired. Against this, a firm may move the worker elsewhere during the delay to perform tasks that have some value to the firm.

Derivation of the optimal contract for any arbitrary duration T yields two of the results already derived for $T = 1$. Because the firm can distinguish between the transfer to workers who are in a good match and the transfer to workers in a bad match, it always insures the worker perfectly against the employment risk: $c^j_1 = c_1$, the flat profile of good matches, holds for all T. But because it cannot monitor the destination of the worker after dismissal and the incentive compatibility constraint for T is not binding, (30) also holds for the last period of the contract for all $T \geq 2$,

$$u'(c^n_T) = G(R_{T+1})u'(c^j_{T+1}) + (1 - G(R_{T+1}))u'(c^u_{T+1}). \tag{32}$$

By a trivial extension of the argument used to demonstrate proposition 3, it can be shown that

$$c^j_{T+1} > c^n_T > c^u_{T+1}. \tag{33}$$

In order to derive the optimal policy during the delay in dismissal, when the incentive-compatibility constraints may bind, I consider first the optimal contract in any period $t \in [3, T-1]$. As already noted, the constraint set defined by (24) and (28) may not be convex. I will again ignore the possibility of nonconvexities and lotteries, and focus instead on the case where the parameters are such that the constraint set is convex. With convexity, the following optimality conditions hold for the contract defined in (19):

$$\beta(1 - G(R_{t+1}))(u'(c^n_t) - u'(c^n_{t+1})) = -\lambda_t u'(c^n_t)u'(c^n_{t+1}) \tag{34}$$

$$\beta G(R_{t+1})(u'(c^n_t) - u'(c^j_{t+1})) = \lambda_t u'(c^n_t)u'(c^j_{t+1}) \tag{35}$$

$$\beta g(R_{t+1})(A^j_t - V(U^n_{t+1})) + \lambda_t = 0. \tag{36}$$

For period 2 the optimality conditions are slightly different because of the added uncertainty about the quality of the match in period 1. Minimization of (18) and (20) subject to (23) and (25), and to the incentive-compatibility constraint (28) for $t = 1$, yields

$$\beta(1 - m)(1 - G(R_2))(u'(c_1) - u'(c^n_2)) = -\lambda_1 u'(c_1)u'(c^n_2) \tag{37}$$

$$\beta(1 - m)G(R_2)(u'(c_1) - u'(c^j_2)) = \lambda_1 u'(c_1)u'(c^j_2) \tag{38}$$

$$\beta(1 - m)g(R_2)(A^j_1 - V(U^n_2) + \lambda_1 = 0. \tag{39}$$

A result that emerges readily from (34)-(36) and (37)-(39) is that if the incentive compatibility constraints are not binding, which in this case requires monitoring of the worker's search effort, $\lambda_t = 0$ and consumption is equalized in all states of nature. The firm can offer first-best insurance against the unemployment risk to its employees who are searching on the job. This result parallels the results in the literature on optimal unemployment insurance. The Appendix shows (in the proof of proposition 4) that without monitoring of search $\lambda_t > 0$, and so, by the concavity of the utility function, (34) and (35) imply

$$c^j_{t+1} > c^n_t > c^n_{t+1}. \tag{40}$$

Proposition 4 *If the value function is a concave function of beginning-of-period assets and workers in poor matches are given notice before dismissal, consumption is flat in all states of nature if the firm can monitor its workers' search effort on the job. If it cannot monitor search effort consumption falls during unsuccessful search on the job, or when the worker is dismissed into unemployment, but rises if the worker is successful in her search and quits to take another job.*

The result on the declining consumption profile during unsuccessful search parallels the result first derived by Shavell and Weiss (1979) for unemployment insurance. However, whereas in their case, because of the absence of a capital market, the result required a declining level of unemployment compensation, when there is unlimited borrowing and lending it can be achieved in a variety of ways, which I discuss briefly in section 5. The key property of the contract offered by the firm is that the lifetime utility of the worker who remains employed in an unproductive job is falling

Proposition 5 *If the firm's cost function is convex, in the case where search on the job cannot be monitored by the firm the lifetime utility of workers on notice of dismissal falls with the duration of employment.*

4.3 Contract length

A contract of the type studied in section (4.1) dominates spot wage offers because severance compensation insures against the risk of an early termination of the job without causing moral hazard or increasing the firm's costs. The question I investigate here is whether the optimal T can be greater than 1.

A delay in dismissal ($T > 1$) may or may not be optimal, partly because of the cost of foregone unemployment compensation and partly because of moral hazard. Consider again the problem studied in section 4.2 for some $T \geq 2$. Inspection of the minimization problem in section 4.2 (the relevant equations are (19)-(20) and (24)-(25)) shows that if the mismatched worker fails to find another job at the beginning of period T, the choices of consumption and savings from the beginning of period T onward are identical to the choices made by an unemployed worker who starts the period with initial assets $V(U_T^n)$ and budget constraint (20). This is a general property of the optimization problem in the last period of the contract: the worker starts the last period of the contract either employed with initial assets A_{T-1}^j or as if she is unemployed with initial assets $V(U_T^n)$ and no income for one period. The firm could dismiss the worker into unemployment at the beginning of period T to take advantage of the compensation b, but it does not do it because its failure to monitor the worker's destination after separation would lead to the breakdown of the contract. The firm would not be able to distinguish between workers who found jobs at the beginning of period T and workers who did not and joined unemployment instead, so it would not be able to differentiate between the payments made in each case, A_{T-1}^j and $V(U_T^n)$ respectively. The tradeoff faced by the firm in the last period of the contract is that it can either set $A_{T-1}^j = V(U_T^n)$ and dismiss the worker to take advantage of the subsidy b in the event of failure in search, or keep the worker and insure her against the risk of the outcome of search by making the transfers A_{T-1}^j and $V(U_T^n)$ contingent on separation.

It follows that if the unemployment subsidy is zero, delaying dismissal for one more period always dominates immediate dismissal. With $b = 0$ the utility levels achieved with noncontingent transfers after separation are in the firm's choice set during a delay in dismissal, but the firm chooses $V(U_T^n) > A_{T-1}^j$ instead.

Proposition 6 *If unemployment income is zero (and there are no other costs of unproductive employment), it is never optimal to dismiss the worker. Workers in unproductive matches are offered severance compensation if they quit and a declining consumption profile if they stay, which induce search on the job and quits.*

Suppose now that instead of zero income, the unemployed enjoy an income which is arbitrarily close to p, their marginal product. Then, trivially (and more formally by an extension of the argument used to prove proposition 1), the worker will never prefer a delay in dismissal over unemployment. Consumption is smoothed completely when the worker can move between employment and unemployment without suffering income loss. By the continuity and monotonicity of value functions with respect to the exogenous income flow during unemployment, it follows that there is a unique and high enough level of unemployment income b^* for which it is optimal to dismiss immediately workers in unproductive jobs.

Proposition 7 *There is a unique value of unemployment income $b^* \in [0, p]$ such that for $b < b^*$ a dismissal delay is offered but for values of $b \geq b^*$ no delay is offered.*

(The proof is trivial and omitted.) More restrictions on the model seem to be required in the derivation of further results. I derive some by analyzing a first-order approximation to the dual of the problem studied so far.

Suppose a worker is in unproductive employment and has reached period $T-1$ without success in her on-the-job search and with inherited contract value $\bar{V}_{T-1} \equiv V(U^n_{T-1})$. Consider now the firm's choice between dismissing this worker at the end of period $T-1$, or offering one more period's delay, and dismissing the worker at the end of period T. The dual of the cost minimization studied in the preceding section is (with some obvious additional notation to distinguish between the two cases):

For dismissal at the end of $T-1$:

$$U^u(\bar{V}_{T-1}) = \beta \max \left\{ u(c^u_{T-1}) + G(R_T)\left[U^j(A^u_{T-1}) - \bar{x}_T\right] + (1 - G(R_T))U^u(A^u_{T-1}) \right\} \tag{41}$$

$$U^u(A^u_{T-1}) = \beta \max \left\{ u(c^u_T) + G(R_{T+1})\left[U^j(A^u_T) - \bar{x}_{T+1}\right] + (1 - G(R_{T+1}))U^u(A^u_T) \right\}. \tag{42}$$

$$\bar{V}_{T-1} \geq \beta(c^u_{T-1} + A^u_{T-1}) \tag{43}$$

$$A^u_{T-1} \geq \beta(c^u_T - b + A^u_T). \tag{44}$$

For dismissal at the end of T :

$$U^n_{T-1} = \beta \max \left\{ u(c^n_{T-1}) + G(R^n_T)\left[U^j(A^j_{T-1}) - \bar{x}^n_T\right] + (1 - G(R^n_T))U^u(V_T) \right\} \tag{45}$$

$$U^u(V_T) = \beta \max \left\{ u(c^n_T) + G(R^n_{T+1})\left[U^j(A^n_T) - \bar{x}^n_{T+1}\right] + (1 - G(R^n_{T+1}))U^u(A^n_T) \right\}. \tag{46}$$

$$\bar{V}_{T-1} \geq \beta(c^n_{T-1} + G(R^n_T)A^j_{T-1} + (1 - G(R^n_T))V_T) \tag{47}$$

$$V_T \geq \beta(c^n_T + A^n_T). \tag{48}$$

In addition, in the second case the incentive compatibility constraint for period $T-1$ has to be observed:

$$R_T^n = U^j(A_{T-1}^j) - U^u(V_T). \tag{49}$$

The conditions satisfied by each of the maximization problems correspond exactly to the ones derived for the optimal contract in the ultimate and prenultimate period of the contract. In the choice of optimal contract length, contract length T is chosen over $T-1$ if and only if $U_{T-1}^n \geq U^u(\bar{V}_{T-1})$, where U_{T-1}^n is the solution to (45) and $U^u(\bar{V}_{T-1})$ is the solution to (41).

Define

$$\Delta_{T-1} \equiv U_{T-1}^n - U^u(\bar{V}_{T-1}) \tag{50}$$

$$\Delta_T \equiv U^u(V_T) - U^u(A_{T-1}^u). \tag{51}$$

Direct substitution from (42) and (46) into (51) and a first-order Taylor approximation around the solutions for the contract that ends in $T-1$ yields

$$\begin{aligned} \beta^{-1}\Delta_T &= u'(c_T^u)(c_T^n - c_T^u) + G(R_{T+1})U^{j\prime}(A_T^u)(A_T^n - A_T^u) \\ &\quad + (1 - G(R_{T+1}))U^{u\prime}(A_T^u)(A_T^n - A_T^u). \end{aligned} \tag{52}$$

Subtracting the budget constraint in (44) from the one in (48) yields

$$\beta(c_T^n - c_T^u + A_T^n - A_T^u) = V_T - A_{T-1}^u - \beta b. \tag{53}$$

Substitution from the first-order condition (7), given the envelope properties $U^{j\prime}(A_T^u) = u'(c_{T+1}^j)$ and $U^{u\prime}(A_T^u) = u'(c_{T+1}^u)$, and from (53) into (52) yields

$$\Delta_T = u'(c_T^u)(V_T - A_{T-1}^u - \beta b). \tag{54}$$

Carrying out a similar Taylor approximation for Δ_{T-1}, we obtain

$$\begin{aligned} \beta^{-1}\Delta_{T-1} &= u'(c_{T-1}^u)(c_{T-1}^n - c_{T-1}^u) + G(R_{T+1})U^{j\prime}(A_{T-1}^u)(A_{T-1}^n - A_{T-1}^u) \\ &\quad + (1 - G(R_{T+1}))\Delta_T. \end{aligned} \tag{55}$$

Taking the difference of the budget constraints (47) and (43), using the result to substitute out of (55) the difference $c_{T-1}^n - c_{T-1}^u$ making use of the first-order condition (7) to substitute out $U^{j\prime}(A_{T-1}^u) - u'(c_{T-1}^u)$ by approximating $G(R_T^n) \approx G(R_T)$, and substituting finally Δ_T from (54) into (55), yields

$$\beta^{-1}\Delta_{T-1} = (1 - G(R_{T+1}))u'(c_T^u)\left(\left(1 - \frac{u'(c_{T-1}^u)}{u'(c_T^u)}\right)(V_T - A_{T-1}^j) - \beta b\right). \tag{56}$$

Therefore, it is optimal to choose contract length T over $T-1$ if

$$\left(1-\frac{u'(c^{u}_{T-1})}{u'(c^{u}_{T})}\right)(V_T - A^{j}_{T-1}) \geq \beta b. \tag{57}$$

By the optimality of choices during unemployment, under strict concavity $c^{u}_{T-1} > c^{u}_{T}$, and by the optimality of contracts, $V_T > A^{j}_{T-1}$, so the left-hand side of (57) is positive. It emerges immediately from (57) that if unemployment income b is zero, it is always optimal to extend the duration of the contract. If b is arbitrarily close to p, the argument of proposition 1 shows that consumption during unemployment does not fall, and so $u'(c^{u}_{T-1}) \approx u'(c^{u}_{T})$, making it unlikely that the inequality in (57) will be satisfied even for small b.

Intuitively, the difference between V_T and A^{j}_{T-1} should be larger when the individual is more risk averse. In the absence of insurance the difference is zero, so an employment contract that offers more insurance should also have a bigger gap between them. Making use of the optimization conditions (34) and (36) to substitute $V_T - A^{j}_{T-1}$ out of (57) we obtain

$$\frac{1-G(R^{n}_{T})}{g(R^{n}_{T})}\frac{1}{u'(c^{n}_{t-1})}\left(1-\frac{u'(c^{u}_{T-1})}{u'(c^{u}_{T})}\right)\left(1-\frac{u'(c^{n}_{T-1})}{u'(c^{n}_{T})}\right) \geq \beta b. \tag{58}$$

For a constant relative risk aversion utility function, the ratio of marginal utilities decreases in the degree of risk aversion for a given the ratio of consumption levels. So other things equal in (58), more risk averse workers will be offered longer duration contracts in the event of poor matches.

An important question about contract length is whether the left-hand side of (58) falls in T, in which case if it is optimal to delay dismissal in period T it is also optimal to delay it in $T-1$. A sufficient condition for this result (although obviously not necessary) is that each term on the left-hand side of (58) falls with T. Although this is almost certain to be satisfied, it is not possible to establish it in general and state the result as a proposition. Intuitively R^{n}_{T} should be rising during search, but unlike the case of unemployed search, the result does not appear to follow from concavity alone. If R^{n}_{T} were rising during search, it would be sufficient for the ratio $\left(1-G(R^{n}_{T})\right)/g(R^{n}_{T})$ to fall in R^{n}_{T}, which is likely to be satisfied by reasonable distributions. The optimal policy during the delay in dismissal requires that consumption fall over time, so $1/u'(c^{n}_{t-1})$ also falls. The relative difference between successive consumption levels should also be either falling or remaining constant during unsuccessful search, as the absolute value of consumption falls, but it cannot be established as a general result. Nevertheless, the fact that $\lim_{t\to\infty} c_t^u = 0$, which is also true for c_t^n, implies that $u'(c_{t-1}^n)$ becomes a very large number as $T \to \infty$, implying that (58) is unlikely to be satisfied for large T, i.e., that if the worker's search search on the job is unsuccessful she will eventually be dismissed into unemployment.

5 Conclusions

This paper has established that in the absence of insurance against income risk, a firm that cannot monitor the search strategy of its workers will offer employment contracts that include severance payments to dismissed employees and, under certain conditions, also delays in dismissal. For these results to hold the firm has to be risk neutral and unemployment insurance has to be less than perfect. An important result is that a delay in dismissal is less likely to be offered the more generous is the compensation to unemployed workers offered by an exogenous unemployment insurance system. Another important result is that the firm offers incentives to employees in unproductive jobs, whose dismissal is delayed for insurance reasons, to search on the job and quit. These incentives take the form of severance compensation in the event of a quit and falling consumption during unsuccessful search. For the firm to achieve the falling consumption profile it has to know the asset position of its workers, a maintained assumption in the paper.

The paper did not examine why severance compensation and dismissal delays should be in legislation and not in private contracts, if optimal. The firm will have an incentive to renege on the contract once the job proves to be unproductive, because it is required to make payments to employees who are quitting. But the conditions under which this incentive can give rise to legislation have not been examined. Nevertheless, if we take the extent to which such measures (commonly referred to as "employment protection") are found in legislation to be a measure of the extent to which private contracts call for them, the model implies an inverse relation across countries between employment protection measures and the generosity of unemployment compensation. Such correlation was found by Boeri et al. (2001) for a sample of (admittedly small) OECD countries.

The method used to derive the results is that of a principal, the firm, minimizing the cost of offering a contract worth a pre-determined utility level to an agent, the worker.

The implementation of the optimal contract was not discussed but it is straightforward to describe some of its main features. The key to the results is twofold. First the worker should be made progressively worse off during unsuccessful on-the-job search, so as to choose a falling consumption profile. The consumption profile of unemployed workers also has this feature, so it can be achieved for employees on delayed dismissal by offering them a wage rate that is less than the wage rate earned by employees in productive jobs. For example, the firm can offer a low wage initially, when the quality of the match is uncertain, and "promote" the workers who prove to be good matches to higher-paying jobs, keeping the rest on the low wage. The wage rate does not have to fall during unsuccessful search on the job.

The second feature of the employment contract is related to its insurance properties and it is the one that implies the optimality of delayed dismissal. The contract should be able to differentiate between the initial asset holdings of workers who quit to join another firm and those who stay with the firm because they are unsuccessful in their search on the job. If the two were identical the worker could achieve the same outcome through the perfect capital market, without insurance. The optimal insurance implied by the contract is that employees who stay because their jobs are productive should start off with lower initial assets but those who leave because of poor matches should start off with higher initial assets. The firm can achieve this outcome by offering severance compensation to unproductive employees who quit, over and above any asset holdings that they may have themselves.

A contract that combines both key features of the firm's optimal contract is one that stipulates an "up or out" policy. Employees who stay because of a good match are given a pay rise. Those who are not in good matches are paid severance compensation when they separate and may also be given notice before dismissal, being kept on the low initial wage during the notice period.

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6 Appendix

6.1 Proofs of Propositions

Proof of Proposition 1. I show first that consumption falls during unemployment. Equations (6) and (3) imply

$$A_{t-1} - A_t = p - b + c_t^u - c_t^j. \tag{59}$$

Let $c_t^u \geq c_{t-1}^u$. Then by (7), $c_t^u \geq c_t^j$ and so

$$A_{t-1} - A_t \geq p - b > 0. \tag{60}$$

The Inada restrictions on the utility function imply that consumption is non-negative in all periods in the horizon, and by (3) this requires $A_t > -p/r$ for all t. (60) yields a contradiction if the consumption path is monotonic because $p > b$ implies that eventually A_t will cross its lower bound. I show that the consumption path is monotonic.

Suppose that there is some t such that $c_{t-1}^u < c_t^u$ and $c_t^u > c_{t+1}^u$. Then from (7), $c_{t+1}^j > c_t^u > c_t^j$. From (3),

$$c_{t+1}^j - c_t^j = r(A_t - A_{t-1}). \tag{61}$$

But (59) implies that $A_t - A_{t-1} < 0$ when $c_t^u > c_t^j$, giving $c_t^j > c_{t+1}^j$, a contradiction. If consumption rises from any period $t-1$ to t, it has to rise from t to $t+1$. In order to avoid the contradiction implied by (60) consumption cannot rise at any time during unemployment.

The employment risk lasts only for one period because of the model's assumptions about job information. The proposition's claim is correct if $c_2^j \geq c_1$. Now, we have shown that $c_2^u > c_3^u$, which by (7) implies $c_3^j > c_2^u$. Suppose $c_2^u > c_2^j$. Then from (59) $A_2 < A_1$, implying $c_3^j < c_2^j$, a contradiction. Therefore $c_2^j > c_2^u$ and so (12) implies $c_2^j > c_1 > c_2^u$.

Proof of Proposition 2. The second part of the proposition follows immediately from the fact that differentiation of (8) and application of the envelope theorem implies

$$\frac{\partial R_t}{\partial A_{t-1}} = u'(c_t^j) - u'(c_t^u) < 0. \tag{62}$$

Thus, if the value of beginning-of-period assets during unemployment falls, the reservation cost rises with duration and so the probability of leaving unemployment, $G(R_t)$, also rises.

Differentiation of the value function (5) for any $t \geq 2$ with respect to A_{t-1} and application of the envelope theorem yields

$$U^{u\prime}(A_{t-1}) - U^{u\prime}(A_t) = G(R_t) \left(U^{j\prime}(A_t) - U^{u\prime}(A_t) \right). \tag{63}$$

The envelope theorem also implies $U^{j\prime}(A_t) - U^{u\prime}(A_t) < 0$. Therefore, $U^{u\prime}(A_{t-1}) - U^{u\prime}(A_t) < 0$ and so if $U^u(A)$ is concave, $A_{t-1} > A_t$

Proof of Proposition 3. Let $\mu < 0$ be the Lagrangian multiplier associated with the constraint in (26) and λ_1 the one for (27) for $T = 1$. The condition that minimizes (21) with respect to R_2 subject to (26) and (27) is

$$\mu(1-m)g(R_2)[U^j(A_1^u) - R_2 - U^u(A_1^u)] + \lambda_1 = 0,$$

which, in view of (27) immediately yields $\lambda_1 = 0$. The conditions that minimize (21) with respect to c_1, A_1 and A_1^u are

$$1 + \mu u'(c_1) = 0 \tag{64}$$

$$1 + \mu U^{j\prime}(A_1) = 0 \tag{65}$$

$$1 + \mu[G(R_2)U^{j\prime}(A_1^u) + (1 - G(R_2))U^{u\prime}(A_1^u)] = 0. \tag{66}$$

Application of the envelope theorem (satisfied because of (2) and (5)) gives the results in the text. Equality between c_1 and c_1^j follows immediately and the other results follow from proposition 1.

Proof of Proposition 4. Consider the maximization of (18) subject to (22) and (28) for any $T \geq 3$, the results for $T = 2$ following immediately. The constraint in (27) can be ignored for the reasons explained in the proof to proposition 3 but there is now a sequential utility constraint

$$U_t^n \geq \bar{U}_t^n \tag{67}$$

in addition to the initial constraint in (22). Intuitively, $\bar{U}_t^n$ is the inherited value of the contract to the worker at the beginning of period t, given the choices made by the firm before t. Let μ_t be the Lagrangian for each constraint in (67). As before, minimization with respect to the controls c_1 and A_1yield (64) and (65), so $c_1 = c_1^j$. Minimization with respect to c_t^n, U_t^n and A_t^j yields

$$1 + \mu_t u'(c_t^n) = 0 \tag{68}$$

$$\beta(1 - G(R_{t+1}))(V'(U_{t+1}^n) + \mu_t) + \lambda_t = 0 \tag{69}$$

$$\beta G(R_{t+1})(1 + \mu_t U^{j\prime}(A_t^j)) - \lambda_t U^{j\prime}(A_t^j) = 0. \tag{70}$$

The envelope theorem gives $V'(U_t^n) = 1/u'(c_t^n)$ and $U^{j\prime}(A_t^j) = u'(c_{t+1}^j)$, which, upon substitution into (69)-(70) yields (34)-(35). Minimization with respect to A_T^u yields

$$1 + \mu_T [G(R_{T+1})U^{j\prime}(A_T^u) + (1 - G(R_{T+1}))U^{u\prime}(A_T^u)] = 0, \tag{71}$$

which can be used to derive (32). Applying the results of proposition 1 gives (33). Finally, minimization with respect to R_t immediately yields (36).

If the firm can monitor search effort the incentive compatibility constraints are not binding and so $\lambda_t = 0$ for all t. It then follows that $c_t^j = c_t^n = c_1$ for all t. If the firm cannot monitor search the proposition's result hinges on the sign of λ_t. I know show that if the incentive compatibility constraints are binding, $\lambda_t > 0$ for all $t = 1, ..., T-1$.

Suppose $\lambda_{T-1} < 0$ and so by (36), $A_{T-1}^j > V(U_T^n)$. From (34) (35), $c_T^n > c_{T-1}^n > c_T^j$ and so by (33) $c_{T+1}^j > c_T^j$ and by (3) $A_T^u > A_{T-1}^j$. To get a contradiction, note that, given that $\beta = 1/(1+r)$, (20) and $A_{T-1}^j > V(U_T^n)$ yield

$$\begin{aligned} A_{T-1}^j &> c_T^n + A_T^u - rA_{T-1}^j \\ &> c_T^j + A_T^u - rA_{T-1}^j \\ &= p + A_T^u > A_T^u. \end{aligned}$$

Therefore $\lambda_{T-1} \geq 0$ and $c_T^j > c_{T-1}^n > c_T^n$.

Working now backwards, suppose $\lambda_{T-2} < 0$. Then $A_{T-2}^j > V(U_{T-1}^n)$, $c_{T-1}^n > c_{T-2}^n > c_{T-1}^j$ and so $c_T^j > c_{T-1}^j$ and $A_{T-1}^j > A_{T-2}^j$. Now, $A_{T-2}^j > V(U_{T-1}^n)$ and (19) yield

$$\begin{aligned} A_{T-2}^j &> c_{T-1}^n - rA_{T-2}^j + (1 - G(R_T))(V(U_T) - A_{T-1}^j) + A_{T-1}^j \\ &> c_{T-1}^j - rA_{T-2}^j + (1 - G(R_T))(V(U_T) - A_{T-1}^j) + A_{T-1}^j \\ &> A_{T-1}^j, \end{aligned}$$

a contradiction. Therefore, $\lambda_{T-1} \geq 0$, and by an analogous argument, $\lambda_t \geq 0$ for all t (with strict inequality if the constraints are binding).

Proof of Proposition 5 From the optimization conditions (68)-(69), and given the envelope property $V'(U^n_t) = 1/u'(c^n_t) = -\mu_t$,

$$\beta(1 - G(R_{t+1}))(V'(U^n_{t+1}) - V'(U^n_t)) = -\lambda_t < 0.$$

Therefore, if $V(.)$ is convex, $U^n_t > U^n_{t+1}$.

Proof of Proposition 6 If unemployment income is zero and the last period of the contract is T, the optimization problem in period $T-1$ in the event of a delay in dismissal is

$$V(U^n_{T-1}) = \beta \min \left\{ c^n_{T-1} + G(R_T)A^j_{T-1} + (1 - G(R_T))V(U^n_T) \right\}, \tag{72}$$

$$V(U^n_T) = \beta \min \{c^n_T + A^u_T\}. \tag{73}$$

$$U^n_{T-1} = \beta \left(u(c^n_{T-1}) + G(R_T) \left(U^j(A^j_{T-1}) - \bar{x}_T \right) + (1 - G(R_T))U^n_T \right), \quad t = 2, ..., T-1 \tag{74}$$

$$U^n_T = \beta \left(u(c^n_T) + G(R_{T+1}) \left(U^j(A^u_T) - \bar{x}_{T+1} \right) + (1 - G(R_{T+1}))U^u(A^u_T) \right). \tag{75}$$

If the worker is unemployed in period $T-1$, the optimization problem is the same again but with the constraint $A^j_{T-1} = V(U^n_T) \equiv A^u_{T-1}$. Therefore a delay in dismissal cannot be worse than unemployment.